arXiv:astro-ph/9503038v2 9 Mar 1995

The Effects of Substructure on Galaxy Cluster Mass Determinations

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Accepted for publication in the *Astrophysical Journal Letters*

Received ______________________; accepted ______________________

ABSTRACT

Although numerous studies of individual galaxy clusters have demonstrated the presence of significant substructure, previous studies of the distribution of masses of galaxy clusters determined from optical observations have failed to explicitly correct for substructure in those systems. In this *Letter* I present the distributions of velocity dispersion, mean separation, and dynamical masses of clusters when substructure is eliminated from the cluster datasets. I also discuss the changes in these distributions because of the substructure correction. Comparing the masses of clusters with central galaxies before and after correction for the presence of substructure reveals a significant change. This change is driven by reductions in the mean separation of galaxies, not by a decrease in the velocity dispersions as has generally been assumed. Correction for substructure reduces most significantly the masses of systems with cool X-ray temperatures, suggesting that the use of a constant linear radius ($1.5h_{100}^{-1}$ Mpc in this study) to determine cluster membership is inappropriate for clusters spanning a range of temperatures and/or morphologies.

1. Introduction

Studies of galaxy cluster dynamics, simulations of structure formation and analytical arguments suggest that we are living in the era of cluster formation. For this reason, the assumption of dynamical equilibrium made in early cluster studies no longer seems valid. Over the last 15 years, the study of substructure in clusters has become vital to the study of clusters in general. From the cosmological point of view, clusters are useful for tracing the large-scale distribution of matter in the Universe, for determining the value of the Hubble constant through the Sun'yaev-Zel'dovich effect and other methods, and for measuring the value of the matter density of the Universe. The presence of substructure implies that non-equilibrium models need to be incorporated into these studies to correctly replicate the behavior of the observed Universe. Similarly, the density of a galaxy's environment appears to have profound effects on the structure and evolution of galaxies. A high frequency of substructure in clusters implies that the environment of any particular galaxy needs to be determined locally, not globally (as first pointed out by Dressler, 1980).

A wide variety of observations in both the optical and X-ray regimes provide convincing evidence that substructure in clusters is common, if not ubiquitous. X-ray imaging observations with the *Einstein* satellite first revealed that complexity in the intracluster medium (ICM) is frequent (Forman et al. 1981; Henry et al. 1981), in contrast to the smooth configurations assumed in early studies (cf. Kent & Gunn 1982; Kent & Sargent 1983). Since then, both optical (Geller & Beers 1982; Baier 1983; Beers et al. 1991; Bird 1993, 1994a,b) and X-ray studies (Jones & Forman 1992; Davis & Mushotzky 1993; Mohr, Fabricant & Geller 1993) have revealed the presence of significant substructure in galaxy clusters.

The physical importance of substructure is less clear. Although most substructure diagnostics are sensitive to deviations on small mass scales (Escalera et al. (1994) claimed

that most of the substructure they detect is on the order of 10% of the total cluster mass), careful analysis reveals that kinematical and dynamical estimates of cluster properties can be severely affected by these apparently low levels of contamination (cf. Beers, Flynn & Gebhardt 1990; Bird 1994a).

In this paper I present an analysis of the dynamical masses of rich galaxy clusters based on the velocities and positions of cluster galaxies. The cluster dataset consists of clusters with central dominant galaxies which have at least 50 measured redshifts (the redshifts are taken from the literature, listed in Table 1 of Bird 1994a). These clusters are limited to morphological types cD (optical) or XD (X-ray); the selection criteria are discussed in more detail in Bird 1994a.

The large observational database permits use of an objective partitioning algorithm called KMM (Ashman, Bird & Zepf 1994) to identify and eliminate substructure "contamination" from the primary subclusters, those that contain the central galaxy. The optical data is supplemented by X-ray temperatures from the literature; these values are published in Bird, Mushotzky & Metzler (1995) along with their sources. The primary purpose of the current work is to show that even in these clusters, long believed to be the most dynamically-evolved systems, the distribution of observed masses is dramatically changed if substructure is objectively eliminated from the cluster datasets. This result is in contrast to that of Biviano et al. (1993) and Escalera et al. (1994), who claimed that the presence of substructure did not affect the results of their optical mass determinations.

In Section 2, I review the dynamical mass estimators and provide the distributions of velocity dispersion, mean galaxy separation and dynamical mass for the cluster dataset, both before and after substructure correction. In addition I quantify the effects of substructure correction on these distributions. I discuss various causes of the change in the cluster mass distribution in Section 3.

2. Dynamical Mass Estimators

For a system in dynamical equilibrium, the Jeans equation relates the kinetic energy of the galaxies to the binding mass of the cluster:

$$-\frac{G\ n_{gal} M_{opt}(r)}{r^2} = \frac{d(n_{gal}\sigma_r^2)}{dr} + \frac{2n_{gal}}{r\sigma_r^2}(1 - \sigma_r^2/\sigma_t^2) \tag{1}$$

(Merritt 1987), where M_{opt} is the optically-determined binding mass, n_{gal} is the galaxy number density, and σ_r and σ_t are the radial and tangential velocity dispersions respectively. By taking the fourth moment of the Jeans equation, Heisler, Tremaine & Bahcall (1985) derive orbital constants for the *projected mass estimator*:

$$M_p = \frac{\xi}{GN} \sum_i v_i^2 R_i \tag{2}$$

where R_i is the projected distance of galaxy i from the D/cD galaxy, and v_i is the velocity of galaxy i with respect to the robust estimator of the velocity location of the cluster, C_{BI} (see Beers, Flynn & Gebhardt 1990 for a complete discussion of the application of robust estimators). The factor ξ is equal to $\frac{32}{\pi}$, assuming a distribution of tracer particles with isotropic orbits moving in a smoothly-distributed gravitational potential.

The assumption of isotropic orbits has not been well-tested, although preliminary X-ray masses from *ROSAT* and *ASCA* suggest that at least in a few clusters, the assumption of isotropy in the galaxy orbits is consistent with the observed data (Mushotzky, 1995). The isotropic orbital constant has been used in most other studies and more importantly, permits a direct comparison with virial theorem mass estimates (for which the assumption of an isotropic orbital distribution is necessary but not explicit in the traditional formulation; see Heisler, Tremaine & Bahcall 1985). The projected mass estimator is statistically preferable to the virial mass estimator, especially for small datasets, and in the majority of clusters yields masses which are consistent with the virial estimator within the errors (Postman, Geller & Huchra 1988).

To correct for the presence of substructure in the cluster datasets, I have used the KMM mixture-modelling algorithm (McLachlan & Basford 1988). KMM is an implementation of a maximum-likelihood technique which assigns each galaxy into a prospective parent population, and evaluates the improvement in fitting a multiple-group model over a single-group model. KMM may be applied to data of any dimensionality; for the cluster data I have simultaneously partitioned the velocity and galaxy position data. In addition to its use in the current study, it has been used as a hypothesis test for the detection of bimodality (Ashman, Bird & Zepf 1994, and references therein). See Bird (1994a) for details of the partitioning.

In any objective partitioning algorithm, verification of the partition is the most difficult and subjective part. For the cluster substructure partitions, we have used several techniques to verify that the KMM algorithm is behaving sensibly. These include comparison with independent X-ray images (Davis et al. 1995; Bird, Davis & Beers 1995) and comparison with the substructure allocation determined by other authors for their optical data (especially Malumuth et al. 1992 and Pinkney et al. 1993). In all cases, the structures identified by independent methods verified the objects identified by KMM.

In Table 1, I present M_p for the clusters in the limited sample defined above, as well as the robust estimator of the velocity dispersion S_{BI} (Beers, Flynn & Gebhardt 1990) and the mean distance of member galaxies from the cluster centroid, $< r_\perp >$. These are determined within a radius of $1.5h_{100}^{-1}$ Mpc, one Abell radius (the effects of this "fixed aperture" calculation are discussed below). Use of the Abell radius as a "fixed aperture" for mass determinations is common; see Biviano et al. (1993) and Beers et al. 1995 for examples. The position of the central galaxy is used as the cluster centroid, following Beers & Tonry (1986). For the clusters in this sample which are included in the Beers & Tonry list, I have verified that the cD and X-ray centroids agree to within 1 arcminute, the

pointing accuracy of the *Einstein* X-ray images. Unprimed quantities are not corrected for the presence of substructure; primed quantities include elimination of galaxies identified by the KMM algorithm as contaminating.

It is important to note that some of these "contaminating" structures are probably themselves gravitationally bound, distinct subgroups in the potential of the central galaxy host subcluster. However, it seems reasonable that low-mass systems undergoing mergers with more massive clusters will be severely disrupted during the interaction. It is inappropriate to apply the dynamical mass estimators, which are predicated on the systems under study being steady state, to the "contaminating" structures, because we have no objective way to distinguish which systems are disturbed and which are not. The central galaxy provides a useful tool by which we can objectively identify the primary cluster. This approach to mass correction differs from that of Biviano et al. (1993), who claimed that substructure did not introduce significant uncertainties into their work, and Escalera et al. (1994), who assumed that all contaminating substructures were gravitationally bound and undisrupted.

In Figure 1, I present the distributions of velocity dispersion, mean distance of a cluster galaxy from the cluster position centroid, and M_p. The visual impression provided by these histograms suggests that the distribution of velocity dispersions has not changed significantly, but that the other distributions have. This subjective impression can be quantified through use of a two-distribution Kolmogorov-Smirnov test (Press et al. 1988). The KS test is useful for distinguishing between parent populations of observational distributions (although it cannot be used to demonstrate that two distributions are the same). The distributions of velocity dispersion S_{BI} are consistent with being drawn from the same parent populations. The before-and-after distributions of $< r_\perp >$ and M_p are strongly inconsistent with each other (at a significance level of 1% in each case).

In Figure 2, I present the cumulative distributions of M_p. The dotted line represents the distribution without substructure correction; the solid line represents the corrected distribution. Note that although the shape of the histograms in Figure 1 has changed substantially, the shape of the cumulative distributions (their slope) is not severely affected by the substructure correction. The normalization, however, is reduced after the inclusion of substructure in the analysis.

Note that contrary to previous impressions, overestimates of dynamical masses do not appear to be caused by overestimates of the cluster velocity dispersions, as is commonly assumed. In any individual galaxy cluster, the velocity dispersion may increase or decrease after KMM is used to eliminate substructure. On the other hand, the mean distance parameter decreases in almost every system in the limited cluster sample. To some extent, this may merely indicate that the use of the "3-σ" velocity filter eliminates more line-of-sight structure than use of a fixed aperture radial cut-off does projected structure. However, it may also suggest that overestimates in dynamical mass are due to the inclusion of galaxies which are not within the virialized core of the cluster. I test this possibility in the next section.

3. What's Going On?

Ashman (1992) argues that for a system to be virialized by the present epoch, it must have a density of 5.8 $\times 10^{13}$ $M_\odot$ Mpc$^{-3} \equiv \rho_{vir}$ (see also Peebles 1993). If we assume that all clusters have similar formation epochs, and that this value is a minimum, we can define a relationship between density, radius and the depth of the gravitational potential well of the cluster. This depth may be estimated using either the velocity dispersion of the cluster galaxies or the temperature of the X-ray emitting gas (cf. Sarazin 1988); here we will use

temperature, because it is unaffected by the substructure corrections applied to the optical data. The proportionality depends on the galaxy cluster being in hydrostatic equilibrium within the core and gravity being the only source of energy for either the galaxies or the gas, in which case:

$$T_X \propto \frac{M}{r_{vir}} \propto \frac{\rho_{vir} r_{vir}^3}{r_{vir}} \propto r_{vir}^2 \tag{3}$$

Therefore we can define the *radius of virialization* r_{vir}, the distance to which the cluster is expected to have reached dynamical equilibrium at the present epoch, by scaling to X-ray temperature T_X. It is convenient to use the values for Coma, which has a gas temperature of $8.4^{+1.1}_{-0.9}$ keV (Watt et al. 1992) and $r_{vir} \approx r_A$, an Abell radius (The & White 1986; Evrard, 1994, private communication). While defining a physical quantity on the basis of one cluster (which does not itself meet the morphological criteria for membership in the current sample) is clearly less than ideal, Coma's X-ray and optical properties are typical of rich clusters and probably do not introduce a large uncertainty in this argument.

Using the values of X-ray temperature for the 21 clusters in the limited sample which have reliable X-ray observations (Bird, Mushotzky & Metzler 1995), we find that the range of radius included by the 1.5 Mpc cutoff is $1-2.5r_{vir}$. For the coolest clusters (A194, A1060, A2052, A2063, A2634, A2670, A3558 and DC1842-63, all with $T < 4$ keV), using a 1.5 Mpc cutoff radius samples portions of the cluster environment well outside the region expected to be virialized. The remainder of the clusters in the limited sample have temperatures between 6 and 9 keV, and values of r_{vir} similar to 1.5 Mpc. The last three columns of Table 1 provide r_{vir}, $M_p(< r_{vir})$ and $M_p'(< r_{vir})$ for the clusters with X-ray temperature determinations. $M_p(< r_{vir})$ and $M_p'(< r_{vir})$ are the projected mass within r_{vir} before and after substructure correction, respectively.

The behaviour of the "before-and-after" distributions of velocity dispersion, mean separation and dynamical mass for the $1.5h^{-1}$ Mpc cutoff helps to quantify this effect. If

the eight coolest clusters are removed, a two-distribution KS fails to distinguish between *any* of the pairs of distributions. That is, the dramatic change in the distributions of mean separation and dynamical mass quantified in Section 2 is due to changes in only the clusters with the lowest X-ray temperatures (and presumably the most shallow gravitational potentials). If we consider the distributions of dynamical mass calculated within r_{vir} rather than within a fixed aperture, the correction for substructure no longer significantly changes the distribution of masses (the two-distribution KS test has a significance level of 27.5%). Even if the assumptions made in the estimate of r_{vir} are incorrect – if clusters don't all form at the same epoch or if their densities are much different than ρ_{vir} – this result suggests that rather than using a fixed aperture for mass determinations (see, for instance, Biviano et al. 1993), use of a physically-motivated radius for each cluster makes the most efficient use of the data and reduces uncertainties due to the presence of substructure. This result depends only weakly on the use of the Coma cluster to define r_{vir}. Note however that even within r_{vir}, the substructure correction can significantly affect the dynamical mass estimators for any particular cluster, as is the case for A2634.

There are a couple of reasons why the present result differs from the earlier work of Biviano et al. (1993) and Escalera et al. (1994). Biviano et al. limit their discussion to masses determined within $0.75h^{-1}$ Mpc of the cluster centroid, in order to reduce the effects of substructure in their mass determinations. This approach works but fails to make the most efficient use of the data; as this paper shows, for the galaxy clusters with the deepest gravitational potentials, galaxies at significantly larger distances are likely to be virialized and therefore suitable for mass determinations. In addition several of the clusters contained in the Biviano et al. study are cool systems, for which this work implies that unvirialized galaxies will be included within $0.75h^{-1}$ Mpc.

Escalera et al. (1994) calculate the total masses of their clusters by summing the masses

of the individual subclusters, and conclude that substructure does not greatly affect total cluster masses. In the case of irregular systems like A548 and A2151, this may be correct (if the subclusters have not yet interacted and remain unperturbed). In regular galaxy clusters like those with dominant central galaxies, any detected substructure, *especially* subclusters with a small fraction of the total mass of the system, are likely to be severely disrupted during their interactions with the "host subcluster." If these subclusters are excluded from the estimate of the total cluster mass, the Escalera et al. results are consistent with those presented here.

No matter how carefully one applies substructure corrections to datasets and how rigorously one determines r_{vir}, dynamical mass estimators based on optical data are subject to potentially large uncertainties. Merritt (1987) pointed out that the value of a virial-type mass estimate may vary by factors of hundreds if the shape of the gravitational potential is unknown, as is the case in most nearby clusters. This work is supported by the simulations results of Carlberg & Dubinski (1991), who find that the "mass-traces-light" assumption commonly used to evaluate optical dynamical masses may greatly underestimate the true mass of the system if the optical velocity dispersions are biased. X-ray mass estimation does not suffer from these uncertainties. It is gratifying to find that in those cases where high quality X-ray data are available, and where large optical datasets make objective identification of substructure straightforward, optical and X-ray mass estimators tend to agree (Davis et al. 1995; Mushotzky 1995). This result suggests that the assumption of mass traces light is probably reliable, as is consistent with preliminary results from gravitational lensing observations (Tyson 1995; Tyson & Fischer 1995).

It is a pleasure to acknowledge Gus Evrard, Chris Metzler and Keith Ashman for their advice and suggestions on the physics of mass determinations and cluster formation. I am especially grateful to Gus for pointing out the importance of using a cluster radius

defined by the X-ray temperature. It is a sincere pleasure to thank the many observers responsible for collecting galaxy redshifts for the last ten years, especially John Huchra, Margaret Geller, Ann Zabludoff, Alan Dressler, Steve Shectman, Eliot Malumuth, Bill Oegerle and John Hill. Their dedication made this analysis possible. I'd also like to thank Eliot Malumuth, Alberto Conti and Sergei Shandarin for their suggestions. Mike West's prompt referee'ing was much appreciated. This work was supported by NSF EPSCoR grant No. OSR-9255223 to the University of Kansas.

Table 1: Kinematical and Dynamical Quantities for the Limited Cluster Sample

	S_{BI}	S'_{BI}	$< r_\perp >$	$< r_\perp >'$	M_{PME}	M'_{PME}	r_{vir}	$\mathrm{M}_p(< r_{vir})$	$\mathrm{M}'_p(< r_{vir})$
	km s^{-1}	km s^{-1}	h^{-1}kpc	h^{-1}kpc	10^{14} M$_\odot$	10^{14} M$_\odot$	h^{-1}kpc	10^{14} M$_\odot$	10^{14} M$_\odot$
A85	810^{+76}_{-80}	810^{+76}_{-80}	773	773	$12.6^{+3.2}_{-3.3}$	$12.6^{+3.2}_{-3.3}$	1330	$11.8^{+2.7}_{-3.9}$	$11.8^{+2.7}_{-3.9}$
A119	862^{+165}_{-140}	1036^{+214}_{-221}	551	271	$8.7^{+1.7}_{-2.5}$	$4.9^{+0.4}_{-0.4}$	1169	$6.8^{+2.0}_{-1.7}$	$4.9^{+0.5}_{-0.5}$
A193	726^{+130}_{-108}	515^{+176}_{-153}	375	217	$5.2^{+2.6}_{-1.6}$	$1.2^{+0.5}_{-0.5}$	1061	$5.2^{+1.8}_{-1.5}$	$1.2^{+0.5}_{-0.6}$
A194	530^{+149}_{-107}	470^{+98}_{-78}	536	420	$8.3^{+1.1}_{-1.1}$	$3.6^{+2.1}_{-2.0}$	732	$1.9^{+0.4}_{-0.6}$	$1.9^{+0.4}_{-0.6}$
A399	1183^{+126}_{-108}	1224^{+131}_{-116}	782	677	$23.4^{+3.1}_{-3.1}$	$21.6^{+5.0}_{-3.7}$	1268	$19.7^{+4.0}_{-4.1}$	$19.9^{+6.4}_{-4.1}$
A401	1141^{+132}_{-101}	785^{+111}_{-81}	678	732	$19.4^{+3.3}_{-4.9}$	$12.3^{+4.2}_{-4.4}$	1518	$19.5^{+3.6}_{-3.5}$	$12.2^{+4.0}_{-3.3}$
A426	1262^{+171}_{-132}	1262^{+171}_{-132}	425	425	$17.2^{+10.2}_{-5.1}$	$17.2^{+10.2}_{-5.1}$	1299	$13.6^{+1.8}_{-2.8}$	$13.6^{+1.8}_{-2.8}$
A496	741^{+96}_{-83}	533^{+86}_{-76}	454	376	$6.1^{+2.1}_{-1.7}$	$2.9^{+1.4}_{-1.3}$	1035	$5.7^{+2.2}_{-0.9}$	$2.3^{+0.5}_{-0.5}$
A754	719^{+143}_{-110}	1079^{+234}_{-243}	711	823	$10.3^{+1.5}_{-1.3}$	$15.1^{+4.8}_{-6.0}$	1527	$10.3^{+1.5}_{-1.3}$	$15.1^{+4.8}_{-6.0}$
A1060	630^{+66}_{-56}	710^{+78}_{-78}	510	316	$3.4^{+0.4}_{-0.6}$	$2.3^{+0.2}_{-0.3}$	940	$3.1^{+0.6}_{-0.3}$	$2.8^{+0.4}_{-0.2}$
A1644	919^{+156}_{-114}	921^{168}_{-141}	743	710	$17.1^{+5.5}_{-7.2}$	$16.9^{+4.6}_{-4.5}$	1048	$14.6^{+5.2}_{-5.0}$	$14.6^{+5.4}_{-5.1}$
A1736	955^{+107}_{-114}	528^{+136}_{-87}	668	529	$13.3^{+3.4}_{-3.5}$	$3.1^{+2.2}_{-1.8}$	1110	$9.7^{+2.4}_{-1.9}$	$3.3^{+2.4}_{-2.1}$
A1795	834^{+142}_{-119}	912^{+192}_{-129}	558	445	$9.5^{+1.8}_{-1.5}$	$9.4^{+2.5}_{-1.4}$	1225	$9.7^{+1.0}_{-1.6}$	$9.6^{+2.3}_{-2.0}$
A1809	782^{+148}_{-125}	851^{+142}_{-154}	473	391	$6.5^{+1.7}_{-1.9}$	$7.2^{+1.8}_{-1.2}$	—	—	—
A1983	646^{+184}_{-129}	532	650	775	$10.2^{+5.1}_{-2.6}$	$3.1^{+1.4}_{-1.3}$	—	—	—
A2052	1404^{+401}_{-348}	714^{+143}_{-148}	553	270	$50.7^{+14.3}_{-10.8}$	$3.4^{+0.6}_{-0.5}$	954	$3.7^{+1.4}_{-1.5}$	$2.7^{+0.6}_{-0.4}$
A2063	827^{+148}_{-119}	706^{+117}_{-109}	459	360	$10.2^{+5.3}_{-2.8}$	$3.9^{+0.5}_{-0.4}$	954	$5.3^{+1.1}_{-1.0}$	$3.6^{+0.1}_{-0.3}$
A2107	684^{+126}_{-104}	577^{+177}_{-127}	396	305	$4.4^{+0.8}_{-0.6}$	$1.8^{+0.7}_{-0.5}$	1061	$4.4^{+0.7}_{-0.6}$	$1.8^{+0.7}_{-0.5}$
A2124	872^{+151}_{-114}	906^{+135}_{-146}	482	263	$7.2^{+2.2}_{-2.0}$	$4.1^{+0.4}_{-0.6}$	—	—	—
A2199	829^{+124}_{-118}	829^{+124}_{-118}	444	444	$6.6^{+1.4}_{-0.9}$	$6.6^{+1.4}_{-0.9}$	—	—	—
A2634	1077^{+212}_{-152}	824^{+142}_{-133}	653	506	$32.0^{+7.5}_{-5.1}$	$10.6^{+1.1}_{-1.1}$	954	$17.0^{+7.5}_{-5.2}$	$7.5^{+3.9}_{-1.7}$
A2670	1037^{+109}_{-81}	786^{+239}_{-203}	512	505	$9.3^{+1.1}_{-1.0}$	$7.1^{+1.8}_{-1.4}$	1022	$8.7^{+1.4}_{-0.9}$	$7.0^{+1.7}_{-2.8}$
A3558	923^{+120}_{-101}	781^{+111}_{-98}	458	377	$9.9^{+2.5}_{-1.9}$	$5.6^{+0.9}_{-1.3}$	1009	$9.9^{+2.5}_{-1.8}$	$5.6^{+1.0}_{-1.3}$
0107-46	1032^{+125}_{-108}	1034^{+130}_{-115}	420	348	$12.4^{+1.0}_{-0.9}$	$11.1^{+2.3}_{-2.2}$	—	—	—
1842-63	522^{+98}_{-82}	565^{+138}_{-117}	695	123	$5.4^{+3.0}_{-1.4}$	$1.0^{+0.4}_{-0.3}$	612	$2.3^{+1.0}_{-0.9}$	$1.0^{+0.4}_{-0.3}$

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Figure Captions

Figure 1: Distributions of velocity dispersion, mean distance of cluster galaxies from the dynamical centroid, and projected masses for the limited cluster sample, before and after the correction for substructure.

Figure 2: The cumulative distribution function of cluster masses. The dotted line is the CDF for masses uncorrected for substructure; the solid line is the CDF for corrected masses.